KINEMATIC AND SINGULARITY ANALYSIS OF PRoM-120 – A PARALLEL ROBOTIC MANIPULATOR WITH 2-PRU/PRS KINEMATIC CHAINS

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Abstract
PRoM-120 was a parallel robotic manipulator which constructed by 2-PRU/PRS kinematic chains. The mobility of this manipulator was 3 DOF, i.e. 1 translational DOF, and 2 rotational DOF (1T2R). The objective of this research was to analyze kinematic, and singularity of PRoM-120 based on its geometrical configuration. Inverse kinematics and velocity analysis were carried out by applying loop vector equation. Meanwhile, all screw systems existed in PRoM-120 were derived using screw theory. Singularity analysis revealed that PRoM-120 had no constraint singularity within its workspace. The kinematic singularity of PRoM-120 showed similar condition to transmission singularity. Three types of both singularities were examined and presented for general condition and selected numerical value of kinematic constants having by PRoM-120. The selected numerical values of those kinematic constants showed only one pose for the singular condition, which was the direct kinematic or the output transmission singularity.

Keywords*: PRoM-120, Screw System, Kinematic Analysis, Singularity Analysis.*

INTRODUCTION

A parallel robotic manipulator, also known parallel manipulator, or parallel mechanism (PM in short), is a type of robotic manipulator which constructed by two or more kinematic chains that connect a fixed base to a platform which forms a closed kinematic chain. This type of manipulator offers several advantages over its counterpart, which is a serial robotic manipulator. These advantages are in term of high payload to weight ratio, better stiffness, better precision, and high acceleration, while a smaller working space is a major drawback [1].

The first parallel manipulator is a 6 degree of freedom (DOF) which had 3 translational motion and 3 rotational motion in the space. PM with the mobility of less than 6, or known as lower DOF PM, becomes more popular more than last three decades [2]–[7]. It was due to the application of PM which required for manipulation of an object in space less than 6 DOF. Using lower DOF PM means that fewer actuators required to be installed on the manipulator base.

Three DOF object manipulation in space can be one of pure translational motion (3T), pure rotational motion (3R), or combined motion, i.e. 1 translational and 2 rotational motion (1T2R) or 2 translational and 1 rotational motion (2T1R). The 3 DOF PM with combined motion has inherent kinematic features which called as parasitic motion and firstly coined by Carreterro et al. [3]. Afterward, Li et al. [4] presented the number and direction of parasitic motion in the 3 DOF PM in relation to different limb arrangements or limb planes. There were seven kinds of PMs or subcategories of limb planes that later be classified into four categories based on the geometrical condition of the PMs. An undesired motion produced by the parasitic motion can reduce manipulation accuracy, quality, and difficulties in calibration [4]. Real-time compensation can be used to counter the problem caused by this parasitic motion.

Singularity analysis was applied to examine the kinematic performance of the PMs. The PMs at singular conditions must be avoided because the kinematic performance of the PMs will degenerate. There are three types of singularity used to evaluate the kinematic performance of the PMs. These three types are a Jacobian-based (kinematic) singularity, a constraint singularity, and a transmission singularity.

The kinematic singularity occurs mathematically when the rank of a Jacobian matrix is deficient [8]. Then, the constraint singularity can be found if the wrench systems

of the PMs become linearly dependent [9]. At last, transmission singularity which proposed by Liu et al. [10] is applied to evaluate the ability of the PMs to transmit motion and force from actuated joints to the platform. Then, Xu et al. [5] revealed a close relationship between transmission singularity and kinematic singularity of the PMs. It was revealed that the inverse, the direct, and the combined kinematic singularity were akin to the input, the output, and the combined transmission singularity, respectively.

The application of PMs with combined motion, especially 1T2R PMs, has found several areas such as machine tools [5], telescope [3], and vibration testing [6], [7]. This paper attempted to apply the PMs in a medical application such as for bone cutting. To accomplish such an application, a PRoM-120 was proposed. PRoM-120 is a parallel robotic manipulator which constructed by 2-PRU/PRS kinematic chains. This manipulator is a 3 DOF PM with combined motions, i.e. 1 translational DOF and 2 rotational DOF.

An analytical approach to PRoM-120 was presented in this paper to obtain its kinematic features and its singularity characteristics. A loop vector equation was applied to determine the inverse kinematic solution and velocity of PRoM-120. Then, screw theory was used to obtain screw systems of PRoM-120. At last, singularity analysis of PRoM-120 was presented to describe its kinematic performance. It was applied to general and selected value of kinematic constants owned by PRoM-120.

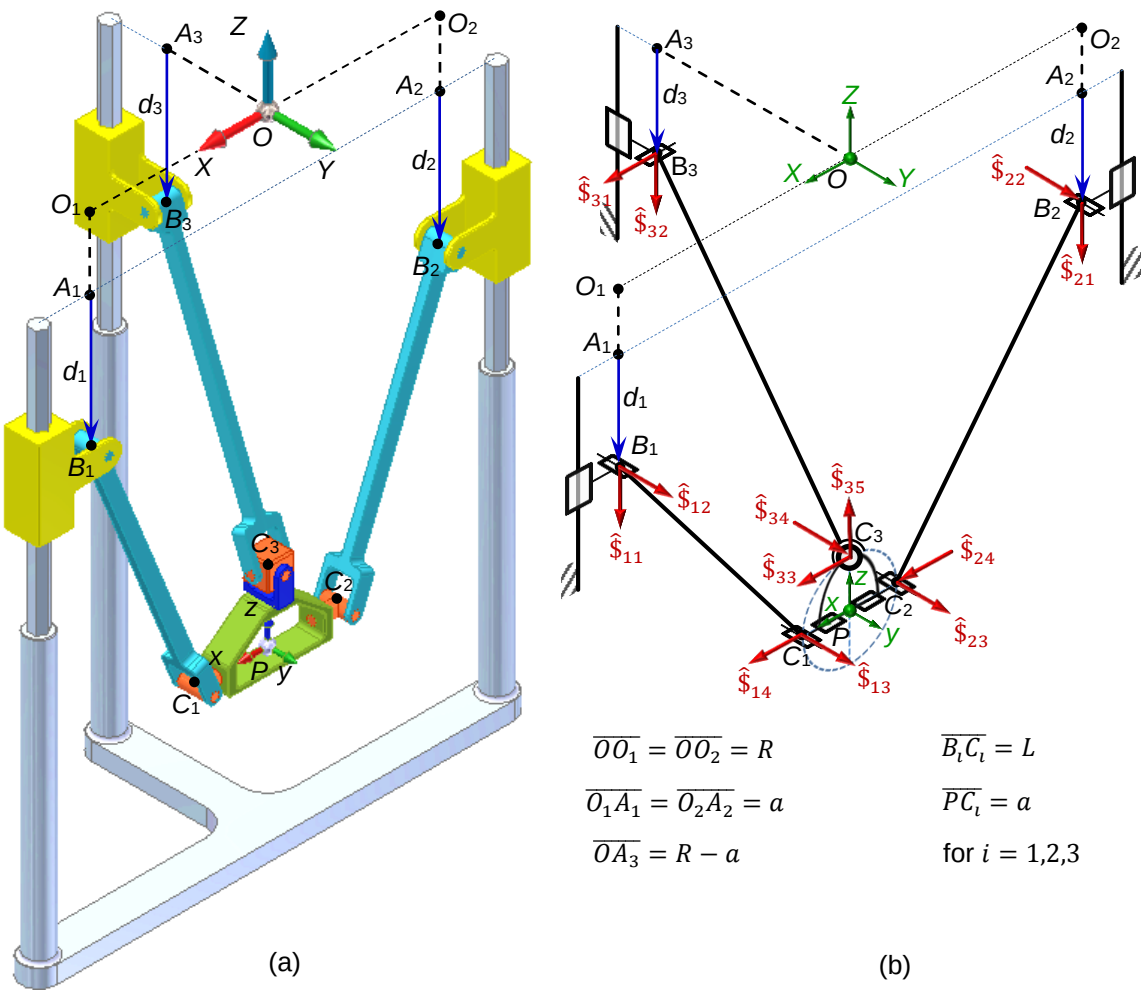

Figure 1. The geometrical architecture of PRoM-120 with 2-PRU/PRS kinematic chain: (a) simplified CAD model and (b) its kinematic diagram with the unit screw on each joint.

METHODS

Configuration of PRoM-120

The kinematic architecture of PRoM-120 was shown in **Figure 1**. This manipulator was constructed by three limbs, one fixed base, and one platform. The first two limbs shared the same kinematic chain which was PRU chain and lied on the same plane, *XZ*-plane. The third limb was composed by PRS kinematic chain and set on the *YZ*-plane.

To describe the kinematic properties of PRoM-120, a fixed reference frame, *O-XYZ*, was assigned to the base. A moving reference frame, *P-xyz*, was set on the platform as depicted in **Figure 1** (b). All prismatic joints located on the fixed base were active joints. The U-joint on the first two limbs could be decomposed into two R-joints that the joint axis intersected with each other. These last R-joints on both limbs had the same axis of the moving frame, which was *x*-axis. The S-joint on the third limb could be represented as three R-joints that intersect with and perpendicular to each other.

Kinematic and Singularity Analysis

The inverse kinematic problem (IKP) of PRoM-120 was obtained by setting the loop vector equation for each limb, that had described in Figure 1. Solving for actuated joint spaces, (d_1; d_2; d_3), gave the inverse kinematic solution of PRoM-120. Meanwhile, the velocity analysis of the manipulator can be obtained by taking the derivative of the inverse kinematic solution with respect to time once.

A screw is a dual vector which constructed by a unit vector along the direction of screw axis, $\mathbf{s}$, and a moment of the unit vector $\mathbf{s}$ about a point of reference, $\mathbf{m}$ [11]. It is stated mathematically as

$$\$ = \rho(\mathbf{s};\quad \mathbf{m}) = \rho(\mathbf{s};\quad \mathbf{r}\times\mathbf{s} + h\mathbf{s}), \tag{1}$$

with $\mathbf{r}$ is the position vector between the unit vector $\mathbf{s}$ towards the point of reference, h denotes the pitch of the screw $\$$, and ρ defines the magnitude of the screw. For a unit screw, ρ is equal to 1, and symbolized by $\hat{\$}$. A reciprocal screw product is a product of the screw $\$$ and another i^{th} screw $\$_i^r$ which satisfies the following relation

$$\$ \circ \$_i^r = \rho\rho_i^r(\mathbf{s}\cdot\mathbf{m}_i^r + \mathbf{s}_i^r\cdot\mathbf{m}) = 0, \tag{2}$$

with $\circ$ represents the reciprocal product of two screws. An interested reader can consult [11] for detail description of the screw theory.

Screw system of PRoM-120 was obtained by applying screw theory. It comprised of a limb twist system (LTS), a limb wrench system (LWS), a platform wrench system (PWS), a platform twist system (PTS), an input twist system (ITS), a transmission wrench system (TWS) and an output twist system (OTS).

The Jacobian-based singularity or kinematic singularity was computed by using a Jacobian matrix which derived from the velocity equation. Meanwhile, motion and force transmission-based singularity was calculated from the last three screw systems presented previously. Also, constraint singularity can be analyzed directly from the PWS.

Finally, a selected case of PRoM-120 was simulated by using its kinematic constants as listed in Table 1. These constants referred to the geometrical architecture of PRoM-120 in Figure 1 and were not the optimized values. Then, the travel distance of each prismatic actuator was set from 0 mm to 200 mm.

Table 1. Kinematic constants of PRoM-120.

No	*Parameter*	*Value*
1	Radius of the base (*R*)	200 mm
2	Length of the link (*L*)	400 mm
3	Radius of the platform (*a*)	80 mm

RESULTS AND DISCUSSIONS

Inverse Kinematic of PRoM-120

PRoM-120 had the orientation capability in two continuous rotations about *x*-axis and *y*-axis that assigned by rotation angles ϑ and ϕ, respectively. The position of point C_i in the fixed frame can be written as

$$\overrightarrow{OC_i} = \overrightarrow{OP} + \mathbf{R}(\vartheta,\phi)\cdot\overrightarrow{PC_i}\,, \tag{3}$$

for i = 1; 2; 3, with $\mathbf{R}(\vartheta,\phi)$ denoted the rotation matrix between the moving frame and the fixed frame, which defined as

$$\mathbf{R}(\vartheta,\phi) = \mathbf{R}(y,\phi)\mathbf{R}(x,\vartheta) = \begin{bmatrix} c_\phi & s_\phi s_\vartheta & c_\vartheta s_\phi \\ 0 & c_\vartheta & -s_\vartheta \\ -s_\phi & c_\phi s_\vartheta & c_\phi c_\vartheta \end{bmatrix}.$$

Then, c_ϑ, s_ϑ, c_ϕ, and s_ϕ were a short notation for cosine and sine of the rotation angles (ϑ and ϕ),

respectively. Meanwhile, the position of point *P* in the fixed frame was $\overrightarrow{OP} = (x \quad y \quad -z)^T$.

The parasitic motion of PRoM-120 can be found by applying the dot product of eq. (3) with a unit vector, $\hat{\mathbf{u}}_i$ (*i* = 1; 2; 3). This unit vector was perpendicular to each limb plane of PRoM-120, i.e. $\hat{\mathbf{u}}_1 = \hat{\mathbf{u}}_2 = (0 \quad 1 \quad 0)^T$, and $\hat{\mathbf{u}}_3 = (1 \quad 0 \quad 0)^T$ and satisfied the condition $\overrightarrow{OC_i} \cdot \hat{\mathbf{u}}_i = 0$. Thus, the condition for the parasitic motion of PRoM-120 was given by

$$\begin{cases} x = -ac_\vartheta s_\phi \\ y = 0 \end{cases}. \tag{4}$$

It showed that the parasitic motion only appeared in the *X*-direction and was regarded as subcategory 2.2 according to [4].

Referring to the geometric description of PRoM-120 as depicted in Figure 1, it can be established the loop vector equation for each limb as given by

$$\overrightarrow{B_iC_i} = -\overrightarrow{OB_i} + \overrightarrow{OC_i}, \tag{5}$$

for *i* = 1; 2; 3, in which $\overrightarrow{B_iC_i}$, $\overrightarrow{OB_i}\left(= \overrightarrow{OA_i} + \overrightarrow{A_iB_i}\right)$, and $\overrightarrow{OC_i}$ denoted the position vector from point B_i to C_i, point *O* to B_i, and point *O* to C_i, respectively. Applying the dot product to eq. (5) produced

$$\overrightarrow{B_iC_i} \cdot \overrightarrow{B_iC_i} = \left(-\overrightarrow{OB_i} + \overrightarrow{OC_i}\right) \cdot \left(-\overrightarrow{OB_i} + \overrightarrow{OC_i}\right), \tag{6}$$

or in the expanded form for each limb as

$$L^2 = k_i^2 + Z_i^2, \tag{7}$$

for *i* = 1; 2; 3, with $k_1 = -\left[R - a\left(c_\phi - c_\vartheta s_\phi\right)\right]$, $k_2 = R - a\left(c_\phi + c_\vartheta s_\phi\right)$, $k_3 = R - a(1 + s_\vartheta)$, $Z_1 = d_1 - z + a\left(1 - s_\phi\right)$, $Z_2 = d_2 - z + a\left(1 + s_\phi\right)$, and $Z_3 = d_3 - z + ac_\phi c_\vartheta$.

An inverse kinematic solution of the manipulator can be obtained by solving the actuated joint spaces (d_1; d_2; d_3) from eq. (7) for the given platform space (z; ϑ; ϕ). It yielded

$$\begin{cases} d_1 = z - a\left(1 - s_\phi\right) + \lambda_1 \\ d_2 = z - a\left(1 + s_\phi\right) + \lambda_2, \\ d_3 = z - ac_\vartheta c_\phi + \lambda_3 \end{cases} \tag{8}$$

with $\lambda_i = -\sqrt{L^2 - k_i^2}$, for *i* = 1; 2; 3.

Velocity of PRoM-120

Velocity equation of the manipulator can be found in a simple way by taking the derivative of eq. (8) with respect to time once. Thus, it can be written mathematically as

$$\begin{cases} \dot{d}_1 = \dot{z} + \frac{\partial \lambda_1}{\partial \vartheta}\dot{\vartheta} + \left(ac_\phi + \frac{\partial \lambda_1}{\partial \phi}\right)\dot{\phi} \\ \dot{d}_2 = \dot{z} + \frac{\partial \lambda_2}{\partial \vartheta}\dot{\vartheta} + \left(-ac_\phi + \frac{\partial \lambda_2}{\partial \phi}\right)\dot{\phi} \\ \dot{d}_3 = \dot{z} + \left(as_\vartheta c_\phi + \frac{\partial \lambda_3}{\partial \vartheta}\right)\dot{\vartheta} + ac_\vartheta s_\phi \dot{\phi} \end{cases}, \tag{9}$$

with $\frac{\partial \lambda_1}{\partial \vartheta} = \frac{ak_1 s_\vartheta s_\phi}{\lambda_1} = \frac{h_{11}}{\lambda_1}$, $\frac{\partial \lambda_1}{\partial \phi} = -\frac{ak_1\left(s_\phi + c_\vartheta c_\phi\right)}{\lambda_1} = \frac{h_{12}}{\lambda_1}$, $\frac{\partial \lambda_2}{\partial \vartheta} = \frac{ak_2 s_\vartheta s_\phi}{\lambda_2} = \frac{h_{21}}{\lambda_2}$, $\frac{\partial \lambda_2}{\partial \phi} = \frac{ak_2\left(s_\phi - c_\vartheta c_\phi\right)}{\lambda_2} = \frac{h_{22}}{\lambda_2}$, and $\frac{\partial \lambda_3}{\partial \vartheta} = -\frac{ak_3 c_\vartheta}{\lambda_3} = \frac{h_{31}}{\lambda_3}$.

By rearranging eq. (9) and presenting them in the matrix form, it gave a simple velocity equation which described as

$$\mathbf{J}_q \dot{\mathbf{q}} = \mathbf{J}_x \dot{\mathbf{x}}, \tag{10a}$$

or

$$\dot{\mathbf{q}} = \mathbf{J}_q^{-1} \mathbf{J}_x \dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{x}}, \tag{10b}$$

with $\mathbf{J}_q$ denoted the Jacobian of the inverse kinematic,

$$\mathbf{J}_q = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix},$$

and $\mathbf{J}_x$ was the Jacobian of the direct kinematic,

$$\mathbf{J}_x = \begin{bmatrix} \lambda_1 & h_{11} & h_{12} + ac_\phi \lambda_1 \\ \lambda_2 & h_{21} & h_{22} - ac_\phi \lambda_2 \\ \lambda_3 & h_{31} + as_\vartheta c_\phi \lambda_3 & ac_\vartheta s_\phi \lambda_3 \end{bmatrix},$$

Meanwhile, Jacobian of the manipulator, $\mathbf{J}$, could be computed by $\mathbf{J}_q^{-1}\mathbf{J}_x$, if $\mathbf{J}_q$ was not a singular matrix. Then, $\dot{\mathbf{q}} = (\dot{d}_1 \quad \dot{d}_2 \quad \dot{d}_3)^T$ and $\dot{\mathbf{x}} = (\dot{z} \quad \dot{\vartheta} \quad \dot{\phi})^T$ were the vector of actuator velocities and platform velocity, respectively.

Screw System of PRoM-120

All screw systems were presented with respect to the fixed frame *O-XYZ*. The screw system which could be constructed by referring to the unit screws as shown in Figure 1 (b) was

a twist system of each limb or the LTS. It represented the motion screw imposed by each joint in every limb. The LTS for the first two limbs was

$$\begin{cases} \hat{\$}_{i1} = (0 \quad 0 \quad 0; \quad 0 \quad 0 \quad -1) \\ \hat{\$}_{i2} = (0 \quad 1 \quad 0; \quad a + d_i \quad 0 \quad R) \\ \hat{\$}_{i3} = (0 \quad 0 \quad 0; \quad p_{i3} \quad 0 \quad r_{i3}) \\ \hat{\$}_{i4} = (c_\phi \quad 0 \quad -s_\phi; \quad 0 \quad q_{14} \quad 0) \end{cases}, \tag{11}$$

for i = 1; 2, with $p_{13} = as_\phi + z$, $p_{23} = -as_\phi + z$, $r_{13} = a(c_\phi - c_\vartheta s_\phi)$, $r_{23} = -a(c_\phi + c_\vartheta s_\phi)$, and $q_{14} = -ac_\vartheta s_\phi^2 - c_\phi z$. Then, the LTS of the third limb was

$$\begin{cases} \hat{\$}_{31} = (0 \quad 0 \quad 0; \quad 0 \quad 0 \quad -1) \\ \hat{\$}_{32} = (1 \quad 0 \quad 0; \quad 0 \quad -d_3 \quad R - a) \\ \hat{\$}_{33} = (1 \quad 0 \quad 0; \quad 0 \quad q_{33} \quad r_{33}) \\ \hat{\$}_{34} = (0 \quad c_\vartheta \quad s_\vartheta; \quad p_{34} \quad 0 \quad 0) \\ \hat{\$}_{35} = (e_{35} \quad -s_\vartheta \quad g_{35}; \quad p_{35} \quad q_{35} \quad r_{35}) \end{cases}, \tag{12}$$

with $q_{33} = ac_\vartheta c_\phi + z$, $r_{33} = as_\vartheta$, $p_{34} = -as_\vartheta^2 - c_\vartheta(ac_\vartheta c_\phi - z)$, $e_{35} = c_\vartheta s_\phi$, $g_{35} = c_\vartheta c_\phi$, $p_{35} = -s_\vartheta z$, $q_{35} = c_\vartheta s_\phi(ac_\vartheta c_\phi - z)$, and $r_{35} = ac_\vartheta s_\vartheta s_\phi$.

Applying reciprocal screw product for each LTS which had obtained previously would produce the LWS. It represented the constraint on each limb. The LWS of the first and the second limb was obtained as

$$\begin{cases} \hat{\$}_{i1}^r = (0 \quad 0 \quad 0; \quad s_\phi \quad 0 \quad c_\phi) \\ \hat{\$}_{i2}^r = (0 \quad c_\phi \quad 0; \quad p_{W2} \quad 0 \quad 0) \end{cases}, \tag{13}$$

for i = 1; 2, and $p_{W2} = ac_\vartheta s_\phi^2 + c_\phi z$. It stated that each limb was constrained by one constraint moment and one constraint force. Meanwhile, the LWS of the third limb was given as

$$\hat{\$}_{31}^r = (1 \quad 0 \quad 0; \quad 0 \quad q_{W3} \quad r_{33}), \tag{14}$$

with $q_{W3} = ac_\vartheta c_\phi - z$. The reciprocal screw given by eq. (14) indicated that the third limb was restrained by one constraint force.

Hence, the PWS can be formed by taking the union of three LWS in eq. (13) and (14) and finding the linearly independent screw. It gave

$$\begin{cases} \hat{\$}_{W1} = (0 \quad 0 \quad 0; \quad s_\phi \quad 0 \quad c_\phi) \\ \hat{\$}_{W2} = (0 \quad c_\phi \quad 0; \quad p_{W2} \quad 0 \quad 0) \\ \hat{\$}_{W3} = (1 \quad 0 \quad 0; \quad 0 \quad q_{W3} \quad r_{33}) \end{cases}. \tag{15}$$

These three wrenches in eq. (15) showed one constraint moment, $\hat{\$}_{W1}$, and two constraint forces, $\hat{\$}_{W2}$ and $\hat{\$}_{W3}$, that restricted motion of the platform.

The PTS of PRoM-120 can be obtained by applying reciprocal screw product to the PWS which gave

$$\begin{cases} \hat{\$}_{M1} = (0 \quad 0 \quad 0; \quad 0 \quad 0 \quad 1) \\ \hat{\$}_{M2} = (0 \quad 1 \quad 0; \quad -q_{W3} \quad 0 \quad 0) \\ \hat{\$}_{M3} = (-c_\phi \quad 0 \quad s_\phi; \quad -r_{33}s_\phi \quad p_{W2} \quad 0) \end{cases}. \tag{16}$$

Eq. (16) described the motion of PRoM-120 or mobility owned by the manipulator. $\hat{\$}_{M1}$, $\hat{\$}_{M2}$, and $\hat{\$}_{M3}$ defined the number of mobility of PRoM-120 which was 3 DOF and the type of motion i.e. 1 translational motion along *Z*-direction, and 2 rotational motion about *Y*-axis and *X*-axis, respectively.

The ITS can be obtained easily from all actuated joints. PRoM-120 was actuated by linear actuators which drove all prismatic joints. Thus, the ITS for each limb was given by

$$\hat{\$}_{Ii} = \hat{\$}_{i1} = (0 \quad 0 \quad 0; \quad 0 \quad 0 \quad -1), \tag{17}$$

for i = 1; 2; 3.

The TWS of each limb can be determined by finding a screw which has to be linearly independent of each passive LTS and the PWS [10]. The passive LTS of the certain limb can be formed from the LTS itself by excluding the ITS of that limb. Taking reciprocal screw product to the passive LTS resulted in reciprocal screws that must not belong to the PWS, eq. (15). There was a unique screw called as the TWS of each limb

$$\begin{cases} \hat{\$}_{T1} = (-k_1 \quad 0 \quad -\lambda_1; \quad 0 \quad q_{T1} \quad 0) \\ \hat{\$}_{T2} = (k_2 \quad 0 \quad \lambda_2; \quad 0 \quad -q_{T2} \quad 0) \\ \hat{\$}_{T3} = (0 \quad k_3 \quad \lambda_3; \quad -p_{T3} \quad 0 \quad 0) \end{cases}, \tag{18}$$

with $q_{T1} = p_{13}k_1 + r_{13}\lambda_1$, $q_{T2} = p_{23}k_2 + r_{23}\lambda_2$, and $p_{T3} = q_{W3}k_3 + r_{33}\lambda_3$. Hence, the TWS of each limb was the transmission force which directed along the link B_iC_i (i = 1; 2; 3).

Finally, the OTS of the certain limb can be found by actuating that limb and locking the others. This condition will produce a new wrench system in the manipulator. As an example, the new wrench system for the first limb can be found by employing the PWS, $[\hat{\$}_{W1},\ \hat{\$}_{W2},\ \hat{\$}_{W3}]$, and the TWS other than that limb, i.e. $[\hat{\$}_{T2}, \hat{\$}_{T3}]$. Also, it can be applied for the other two limbs as well. Thus, the new wrench system, $\mathbf{W}_i$ (i = 1; 2; 3) can be written as

$$\begin{cases} \mathbf{W}_1 = [\hat{\$}_{W1},\ \hat{\$}_{W2},\ \hat{\$}_{W3},\ \hat{\$}_{T2},\ \hat{\$}_{T3}] \\ \mathbf{W}_2 = [\hat{\$}_{W1},\ \hat{\$}_{W2},\ \hat{\$}_{W3},\ \hat{\$}_{T1},\ \hat{\$}_{T3}]. \\ \mathbf{W}_3 = [\hat{\$}_{W1},\ \hat{\$}_{W2},\ \hat{\$}_{W3},\ \hat{\$}_{T1},\ \hat{\$}_{T2}] \end{cases} \tag{19}$$

The OTS of the first, the second and the third limb were obtained by applying reciprocal screw product, eq. (2), to $\mathbf{W}_i$ (i = 1; 2; 3), respectively. It gave

$$\hat{\$}_{Oi} = (e_{Oi} \quad f_{Oi} \quad g_{Oi};\quad p_{Oi} \quad q_{Oi} \quad r_{Oi}), \tag{20}$$

with $e_{O1} = -c_\phi \lambda_3 \mu_1$, $f_{O1} = c_\phi s_\vartheta \lambda_2 \lambda_3 + c_\vartheta \lambda_2 k_3 + s_\phi s_\vartheta k_2 \lambda_3$, $g_{O1} = s_\phi \lambda_3 \mu_1$, $p_{O1} = -a s_\phi s_\vartheta \lambda_3 \mu_1 - q_{W3} f_{O1}$, $q_{O1} = p_{W2} \lambda_3 \mu_1$, $r_{O1} = -a(c_\phi s_\vartheta \lambda_3 + c_\vartheta k_3)\mu_1$, $\mu_1 = (s_\phi - c_\phi c_\vartheta)k_2 + (c_\phi + c_\vartheta s_\phi)\lambda_2$, $e_{O2} = -c_\phi \lambda_3 \mu_2$, $f_{O2} = -s_\phi s_\vartheta k_1 \lambda_3 - (c_\vartheta k_3 + c_\phi s_\vartheta \lambda_3)\lambda_1$, $g_{O2} = s_\phi \lambda_3 \mu_2$, $p_{O2} = -a s_\phi s_\vartheta \lambda_3 \mu_2 - q_{W3} f_{O2}$, $q_{O2} = p_{W2} \lambda_3 \mu_2$, $r_{O2} = -a(c_\phi s_\vartheta \lambda_3 + c_\vartheta k_3)\mu_2$, $\mu_2 = (s_\phi + c_\phi c_\vartheta)k_1 + (c_\phi - c_\vartheta s_\phi)\lambda_1$, $e_{O3} = -c_\phi \mu_3$, $f_{O3} = s_\vartheta s_\phi(-k_1 \lambda_2 + \lambda_1 k_2)$, $g_{O3} = s_\phi \mu_3$, $p_{O3} = -s_\vartheta s_\phi[2a c_\phi \lambda_1 \lambda_2 + (a s_\phi + z)k_1 \lambda_2 + (a s_\phi - z)\lambda_1 k_2]$, $q_{O3} = p_{W2} \mu_3$, $r_{O3} = a s_\vartheta s_\phi[2 s_\phi k_1 k_2 + k_1 \lambda_2 (c_\phi + c_\vartheta s_\phi) + \lambda_1 k_2 (c_\phi - c_\vartheta s_\phi)]$, and $\mu_3 = 2 c_\phi \lambda_1 \lambda_2 + c_\phi c_\vartheta (k_1 \lambda_2 - \lambda_1 k_2) + s_\phi (k_1 \lambda_2 + \lambda_1 k_2)$.

Singularity of PRoM-120

a) Constraint singularity

The constraint singularity occurred if constraint wrench screws in the manipulator became linearly dependent [9]. For PRoM-120, it can be checked from each screw in the PWS, eq. (15), whether linearly dependent or not. The preliminary knowledge was the condition of two constraint forces, $(\hat{\$}_{W2}, \hat{\$}_{W3})$, that perpendicular to each other. These two wrenches were linearly independent. Then, the thorough examination evinced that each screw in the PWS was always linearly independent to each other. It meant that rathe nk of the PWS was always three. Therefore, PRoM-120 did not have constraint singularity.

b) Inverse kinematic and input transmission singularity

The inverse kinematic singularity (IKS) of PRoM-120 occurs if and only if

$$|\mathbf{J}_q| = \lambda_1 \lambda_2 \lambda_3 = 0, \tag{21}$$

for the non-deficient rank of $\mathbf{J}_x$. Eq. (21) can be fulfilled if one of λ_1, λ_2, or λ_3 was equal to zero. At the IKS, the manipulator will lose one or more DOF which enables it to resist forces and moments while the actuators are locked.

The input transmission singularity exists if any certain motion imposed by the ith actuator cannot be transmitted by the TWS of that limb [10]. It was defined as

$$\hat{\$}_{Ii} \circ \hat{\$}_{Ti} = \lambda_i = 0, \tag{22}$$

for i = 1; 2; 3. Reciprocal product of $\hat{\$}_{Ii}$ and $\hat{\$}_{Ti}$ in eq. (22) expressed that the twist screw of ith lithe mb was perpendicular to the transmission wrench screw of the ith limb. In other hand, it signified that the link B_iC_i was parallel to the XY-plane. Therefore, the singular condition given by the IKS was akin to condition of the input transmission singularity, as revealed previously by [5].

The perpendicularity condition for each limb was satisfied by the following condition

$$\begin{cases} L = |R - a(c_\phi - c_\vartheta s_\phi)|; \text{ for } \lambda_1 = 0 \\ L = |R - a(c_\phi + c_\vartheta s_\phi)|; \text{ for } \lambda_2 = 0 \\ L = |R - a(1 + s_\vartheta)|; \quad \text{for } \lambda_3 = 0 \end{cases}. \tag{23}$$

Referring to eq. (23) can be obtained a condition for a length of the link B_iC_i (i = 1; 2; 3) which was $L \geq R - a$. Figure 2 presented three configurations of PRoM-120 at this type of singularity, as expressed mathematically in eq. (23) for L was slight greater than $R - a$.

Thus, PRoM-120 with selected numerical values of its kinematic constants as given by Table 1 and the selected travel distance of the prismatic joints did not experience the IKS and the input transmission singularity.

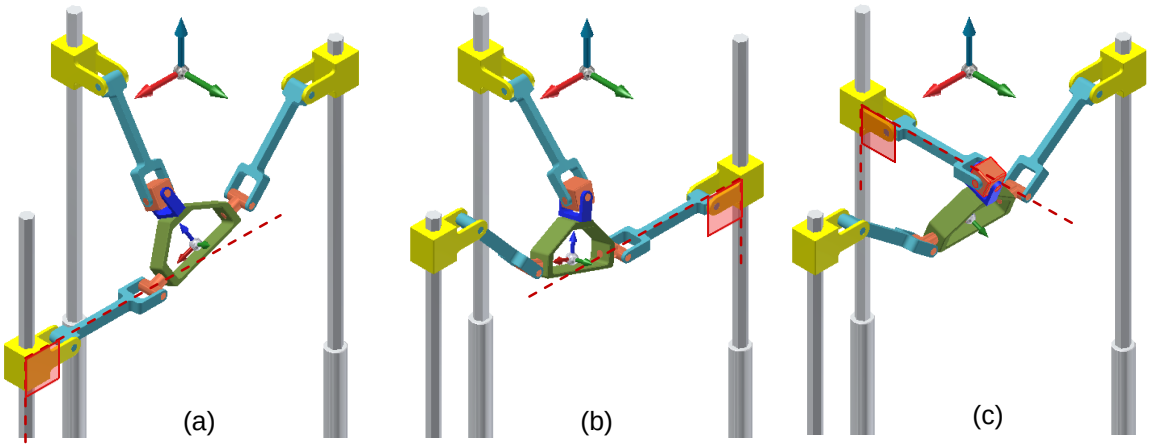

Figure 2. PRoM-120 configurations at the IKS or the input transmission singularity: (a) link B_1C_1, (b) link B_2C_2, and (c) link B_3C_3 were perpendicular to the first, second, and third prismatic joint, respectively.

c) Direct kinematic and output transmission singularity

The direct kinematic singularity (DKS) of PRoM-120 occurs if $|\mathbf{J}_x| = 0$ for the non-deficient rank of $\mathbf{J}_q$, or

$$\begin{vmatrix} \lambda_1 & h_{11} & h_{12} + ac_\phi\lambda_1 \\ \lambda_2 & h_{21} & h_{22} - ac_\phi\lambda_2 \\ \lambda_3 & h_{31} + as_\vartheta c_\phi\lambda_3 & ac_\vartheta s_\phi\lambda_3 \end{vmatrix} = 0. \quad (24)$$

In the singular condition, one or more DOF can be gained by the manipulator which enables it to perform infinitesimal motion even though no motion provided by the actuators. It also means that forces or moments cannot be held by the manipulator at this condition. Referring to eq. (24), it was hard to find the possibilities of the DKS. Alternatively, one can found from each column of $|\mathbf{J}_x|$, in which every element of each column must be zero, i.e.$\lambda_1 = 0$; $\lambda_2 = 0$; $\lambda_3 = 0$, or $h_{11} = 0$; $h_{21} = 0$; $h_{31} + as_\vartheta c_\phi\lambda_3 = 0$, or $h_{12} + ac_\phi\lambda_1 = 0$; $h_{22} - ac_\phi\lambda_2 = 0$; $ac_\vartheta s_\phi\lambda_3 = 0$.

Then, the output transmission singularity occurs if any certain transmission wrench of the ith limb $\hat{\$}_{Ti}$ cannot transmit power from the actuator of the ith limb to provide motion of the platform which represented by $\hat{\$}_{Oi}$ [10]. It was defined as

$$\hat{\$}_{Ti} \circ \hat{\$}_{Oi} = 0, \quad (25)$$

for i = 1; 2; 3. The physical meaning behind this singular condition for each limb was a condition when the link B_iC_i was in the plane to the platform plane. The platform plane was a plane formed by the point C_1, C_2, and C_3. In other words, the link B_iC_i (i = 1; 2; 3) was either in fully extended or fully folded condition with the platform. In reality, the last cannot be reached because of joint limitation and rigid body interferences among certain components.

When the link B_iC_i (i = 1; 2; 3) lying on the same plane with the platform plane, PRoM-120 could undergo some infinitesimal motion while the actuators were locked. Hence, it meant that the output transmission singularity was similar to the DKS which emerged previously by [5]. Figure 3 presented only three configurations of PRoM-120 when experiencing this type of singularity for L was slight greater than $R - a$.

PRoM-120 with selected numerical values of its kinematic constants, as listed in Table 1, and the selected travel distance of the prismatic joints gave one configuration for this type of singularity. This configuration was the link B_3C_3 lying on the same plane with the platform plane.

d) The combined singularity of the kinematic and the transmission singularity

The combined kinematic singularity (CKS) arise when the inverse and the direct kinematic singularity occur concurrently. Also, it is the same with the combined transmission singularity, where both input and output transmission singularity coexisted. It has been shown by [5] for Tex3 parallel manipulator.

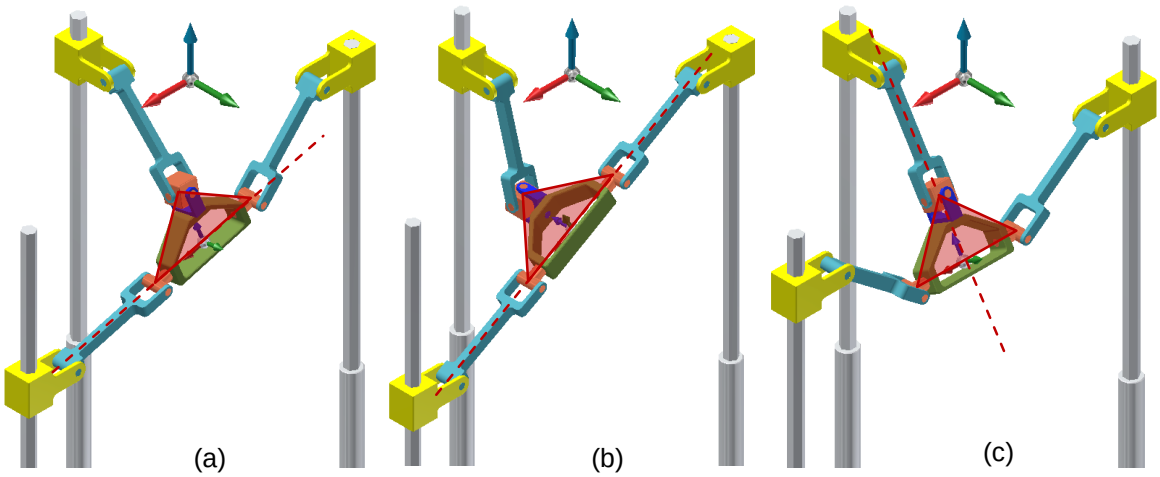

Figure 3. PRoM-120 configurations at the DKS or the output transmission singularity: (a) link B_1C_1, (b) both link B_1C_1 and B_2C_2, and (c) link B_3C_3 were in the plane to the platform plane, respectively.

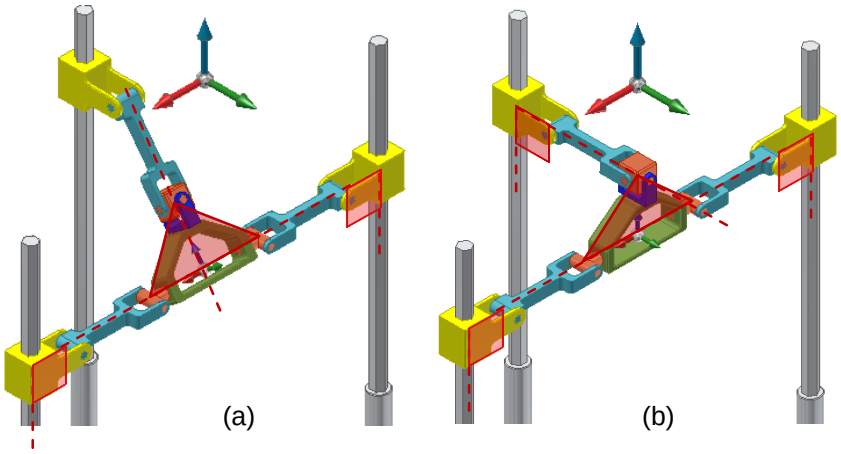

Figure 4. Two PRoM-120 configurations at the CKS or the combined transmission singularity: the first two links perpendicular to the first two prismatic joints and parallel to the platform plane while (a) the third links parallel to the platform plane and (b) the third limb perpendicular to the third prismatic joint.

This type of singularity happened for special architectural dimension, for PRoM-120, it was $L = R - a$. Figure 4 depicted two configurations of the these combined singularities for the condition of $L = R - a$.

The combined singularity did not occur for PRoM-120 with selected numerical values of its kinematic constants as listed previously in Table 1 and the selected travel distance of the prismatic joints. It was due to the fact that PRoM-120 did not possess the IKS and the input transmission singularity for the given numerical values of its kinematic constants.

CONCLUSIONS

Kinematic and singularity analysis of PRoM-120 had been presented in this paper. There were five items that could be outlined as the conclusions, that were

1. Kinematic analysis of PRoM-120, i.e. the inverse kinematic problem and the velocity equation, was a simple equation in closed form solution.
2. PRoM-120 did not possess constraint singularity because wrench system (the PWS) was always linearly independent or having rank equal to the number of wrench system, which was 3.

3. It was evinced and proved that the kinematic singularity had a close relationship with the transmission singularity. The IKS, the DKS, and the CKS were similar to the input, the output, and combined transmission singularity, respectively.
4. The IKS of PRoM-120 existed when one of the links perpendicular to each prismatic joint. Then, the DKS of PRoM-120 occurred when one of the links was in the plane with the platform plane. Finally, the CKS happened if both the IKS and DKS occurred at the same time.
5. It was shown that only one singular condition known for PRoM-120 with the selected numerical values as given in Table 1, i.e. the third link was in the parallel to the platform plane. This singular condition fell to the DKS or the output transmission singularity.

Design and optimization of the kinematic constants had by PRoM-120 will be discussed in the future study.

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